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**A.E.M. 557**

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2.1.2. μ Ball.....	. 17
2.1.3 μ	. 20
2.2	
μ	. 22
2.3	
.....	. 23
3 μ μ	. 26
4 μ	
4.1	. 33
4.2 μ	. 33
4.3 μ	. 34
4.4	. 34
4.5 μ	. 38
4.6 μ	. 41
4.6.1	. 41
4.6.2 μμ	. 52
4.6.3 μ μ	. 54
4.6.4 μ	. 61
4.6.5 -	. 64
4.6.6 μ μ	. 70
.....	. 70
5 μ μ	. 72
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## **Abstract**

Teachers nowadays are called upon to organize the teaching of mathematics focusing on the student himself, using teaching methods that activate him, in order to lead to conceptual understanding rather than sterile procedural knowledge. Many researchers have tried to approach the knowledge that teachers need to have in order to be able to teach appropriately towards this direction.

This dissertation explores the characteristics of content knowledge as well as the pedagogical knowledge of primary school teachers in mathematics. For the purposes of the study, a questionnaire of 17 questions was constructed, nine of which concerned content knowledge and eight pedagogical knowledge in mathematics, which was answered by 214 in-service and prospective teachers.

The results highlight significant teacher shortages related to both content knowledge and pedagogical knowledge. Participants showed weaknesses in the understanding of basic mathematical concepts, such as fractions, geometric shapes and decimal numbers, but also in mathematical processes as in the subtraction algorithm. In addition, basic difficulties were identified in the teachers' understanding of how students approach knowledge, in defining and interpreting students' misconceptions and in selecting the appropriate teaching approach to a mathematical concept.

**Key words:** knowledge of mathematical content, pedagogical knowledge of mathematics, primary school teachers

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(Papert & Harel, 1991). μ

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Ball (2008),

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Rowland (2005), μ μ





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 Council of Teachers (1989)  
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(Shulman, 1987).

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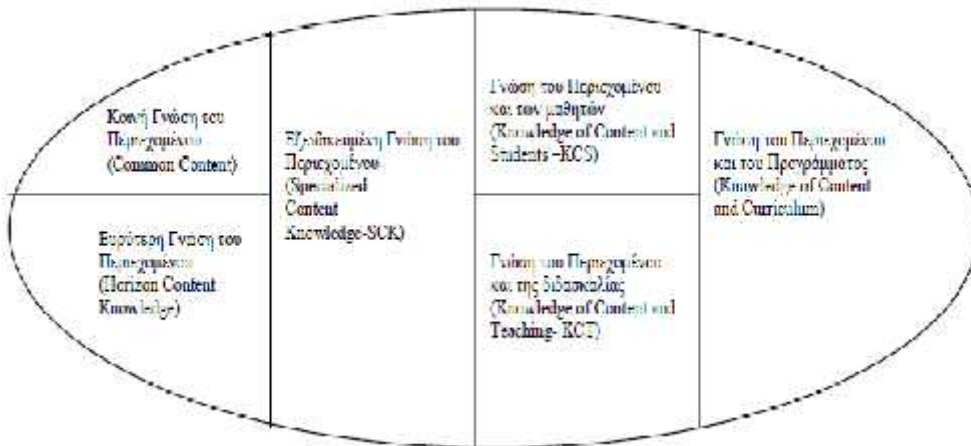
2.1.2. Ball

Shulman

(Ball, Thames & Phelps, 2008).

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Ball et al. (2009a)

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et al., 2009a).

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 μ (Pedagogical Content Knowledge)  
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 (knowledge of content and teaching, KCT) ) μμ  
 (knowledge of curriculum) (Ball et al., 2009a).  
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<< >> (Knowledge quartet), Rowland, Huckstep Twaites (2005),

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 56%.  
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 (2012) μ 294 μ μ  
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 & Christiansen (2015) 19 , Maniraho  
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μ μ (Shulman, 1987)

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μ

		%	
	61	28,5	28,5
	153	71,5	100,0
< 1	127	59,3	59,3
1-5	11	5,1	64,5
6-15	27	12,6	77,1
16-25	31	14,5	91,6
25	13	6,1	97,7
	5	2,3	100,0
/ μ μ	2	0,9	0,9
/ μ μ	6	2,8	3,7
/ μ	21	9,8	13,6
/	185	86,4	100,0
/ μ	2	0,9	0,9
/	212	99,1	100,0
18-22	113	52,8	52,8
23-33	41	19,2	72,0
34-44	17	7,9	79,9
45	42	19,6	99,5
	1	0,5	100,0
	214	100,0	

μμ (214), 153 (71,5%),  
 61 (28,5%). μ  
 μ (127 μ , 59,3%), 31 μ  
 16-25 (14,5%), 27 6-15 (12,6%), 13  
 25 (11,4%), 11  
 (5,1%), 5 .  
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 μ μ ,  
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 μ μ (86,4%)-  
 μ , 21 μ μ  
 μ (9,8%), 6 μ μ μ μ (2,8%),  
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μ , 113 -  
 - 18 22 (52,8%), μ  
 μ μ μ , 42  
 45 (19,6%), 41 23-33 (19,2%), 17 μ 34-  
 45 (7,9%), 1 μ .  
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 μ , 126 (58,9%),  
 . 34 μ (15,9%)  
 , 29 (13,6%)  
 & , 15 (7%)  
 , 10 (4,7%)



## 4.6

μ

### 4.6.1

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 μμ μ SPSS ( Statistical Package  
 for the Social Sciences).

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### 4.2

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μμ

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μ

		%
3/9	73	34,1
1/2	9	4,2
3/8	18	8,4
	111	51,9
	3	1,4
	214	100,0

μ

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(34,1%)

( 3/9).

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(51,9%).

### 4.3

μ

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0,80

0,90

	( )					
	μ					
	119 (55,6%)	22 (10,3%)	54 (25,2%)	0 (0%)	19 (8,9%)	214 (100%)
	137 (64%)	16 (7,5%)	25 (11,7%)	2 (0,9%)	34 (15,9%)	214 (100%)
μ	20 (9,3%)	99 (46,3%)	61 (28,5%)	4 (1,9%)	30 (14%)	214 (100%)
	27 (12,6%)	42 (19,6%)	104 (48,6%)	2 (0,9%)	39 (18,2%)	214 (100%)
	61 (28,5%)	24 (11,2%)	87 (40,7%)	5 (2,3%)	37 (17,3%)	214 (100%)

0,90.  $\mu$   $\mu$   $\mu$   $\mu$  0,80  
 $\mu$  ,  $\mu$   $\mu$  (55,6%)  
 ( ).  $\mu$  (25,2%).  
 $\mu$   $\mu$  (64%)  $\mu$  ,  $\mu$   $\mu$   
 ( ),  $\mu$   
 8%. , 15,9%  
 $\mu$  .  
 -  $\mu$   $\mu$   $\mu$   $\mu$   
 $\mu$  0,80 0,90 (46,3%) ,  
 $\mu$  .  
 $\mu$  (28,5%).  
 $\mu$   $\mu$   $\mu$  .  
 (14%)  $\mu$  .  
 $\mu$  -  $\mu$   $\mu$   $\mu$   
 $\mu$  .  $\mu$  (48,6%)  
 ,  $\mu$  .  
 $\mu\mu$   $\mu$  (19,6%) ,  
 $\mu$   $\mu$  , (12,6%)  
 $\mu$   $\mu$   
 (0,9%). (18,2%) .  
 -  $\mu$   $\mu$   $\mu$   
 $\mu$   $\mu$  (11,2%) ,  $\mu$   $\mu$   $\mu$   
 $\mu$  .  $\mu$  (40,7%)  
 $\mu$  , (28,5%)  
 $\mu$   $\mu$  0,8 0,9 .  
 (2,3%) (17,3%).

4.4

μ

		%
μ	28	13,1
μ	10	4,7
μ	69	32,2
μ	102	47,7
	5	2,3
	214	100,0

μ (47,7%) ( μ 8).

μ (32,2%) μ

μ

6,15 .

4.5

μ 0,125

	( )			
	μ			
	64 (29,9%)	141 (65,5%)	9 (4,2%)	214 (100%)
	146 (68,2%)	59 (27,6%)	9 (4,2%)	214 (100%)
0,125 %	35 (16,4%)	171 (79,9%)	8 (3,7%)	214 (100%)
	68 (31,8%)	136 (63,6%)	10 (4,7%)	214 (100%)
	130 (60,7%)	70 (32,7%)	14 (6,5%)	214 (100%)

μ 0,125. μ (65,9%), « (68,2%), «0,125%» μ (79,9%), « μ (63,6%) « (60,7%).



**4.8**

		%
0	149	69,6
0	14	6,5
0	12	5,6
2	37	17,3
	2	0,9
	214	100,0

μ , μ μ , μ (17,3%) , μ μ μ 2. μ μ μ 0 (69,6%). μ μ μ μ μ μ (6,5% 5,6% ).

**4.9**

	( )		
		μ	
μ	119 (93%)	15 (7%)	214 (100%)
μ	77 (36%)	137 (64%)	214 (100%)
μ	181 (84,6%)	33 (15,4%)	214 (100%)
μ	182 (85%)	32 (15%)	214 (100%)
μ	161 (75,2%)	53 (24,8%)	214 (100%)

μ μ . μ μ : 93%, 84,6%, 85% 75,2% . μ 1 3 μ (36%).

4.10

/ μ

	( )			
		μ		
μμ	166 (77,6%)	47 (22%)	1 (0,5%)	214 (100%)
μμ	62 (29%)	152 (71%)	0 (0%)	214 (100%)
μ μμ	171 (79,9%)	41 (19,2%)	2 (0,9%)	214 (100%)
μμ	23 (10,7%)	187 (87,4%)	4 (1,9%)	214 (100%)
μ	118 (55,1%)	95 (44,4%)	0 (0%)	214 (100%)
μ ,	60 (28%)	153 (71,5%)	1 (0,5%)	214 (100%)
μ μμ	90 (42,1%)	122 (57%)	2 (0,9%)	214 (100%)

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μ μ μ ,  
μ . - μ μ  
μμ (77,6%) , μ μ (22%) μ  
, μ - μ (71%)  
,  
μ (29%). - μ ,  
(79,9%) , 1 5  
(19,2%) . - μ  
μ (87,4%), μ μ (10,7%)  
μ . μ - μ  
μ μ (55,1%) ,  
μ (44,4%). - μ μ  
(71,5%) , μ  
(28%). - μ ,  
μ (57%), μ (42,1%).

**4.11**

		( )		
μ	μ	41 (19,2%)	173 (80,8%)	214 (100%)
2	20	18 (8,4%)	196 (91,6%)	214 (100%)
		185 (86,4%)	29 (13,6%)	214 (100%)
2	μ	10 (4,7%)	204 (95,3%)	214 (100%)
0	μ	48 (22,4%)	166 (77,6%)	214 (100%)

μ

μ μ μ

μ . μ μ μ , μ μ

(86,4%) μ

μ . μ μ

0 μ (22,4%),

μ μ 2720 (19,2%) 2 20 (8,4%).

μ μ

μ 2 (4,7%).

**4.12**

		( )		
μ	μ	26 (12,1%)	188 (87,9%)	214 (100%)
μ	μ	15 (7%)	199 (93%)	214 (100%)
μ	μ	82 (38,3%)	132 (61,7%)	214 (100%)
μ	μ	84 (39,3%)	130 (60,7%)	214 (100%)

μ μ μ  
 μ μ (38,3%)  
 μ μ μ  
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 μ μ (39,3%).  
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 (7%).

#### 4.13

μ μ

	( )		
μ 1	31 (14,5%)	183 (85,5%)	214 (100%)
μ 2	105 (49,1%)	109 (50,9%)	214 (100%)
μ 3	13 (6,1%)	201 (93,9%)	214 (100%)
μ 4	53 (24,8%)	161 (75,2%)	214 (100%)

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 μ μ μ (49,1%).  
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 (24,8%), μ μ  
 (14,5% 6,1% ). μ , μ  
 (28,5%).





4.15

	μ	%
	30	14
	6	2,8
	20	9,3
μ	84	39,3
μ μ	12	5,6
	62	29
	214	100

μ  
 μ , μ  
 μ . μ  
 . μ ,  
 μ μ  
 (39,3%). , μ  
 (14%).  
 μ (9,3%),  
 μ μ μ (5,6%)  
 (2,8%). μ  
 (29%) .

4.16

	μ		
	( )		
	60 (28%)	157 (72%)	214 (100%)
	42 (19,6%)	172 (80,4%)	214 (100%)
0 ,	36 (16,8%)	178 (83,2%)	214 (100%)
μ μ μ	148 (69,2%)	66 (30,8%)	214 (100%)

μ μ μ  
 μ , μ μ  
 μ μ  
 μμ (69,2%) μ .

μ μ μ μ .  
 ,  
 μ (28%)  
 μ (19,6%).  
 (16,8%) μ

**4.17**

μ

	( )		
μ	22 (10,3%)	192 (89,7%)	214 (100%)
μ μ μ	157 (73,4%)	57 (26,6%)	214 (100%)
μ , μ	43 (20,1%)	171 (79,9%)	214 (100%)
μ μ	26 (12,1%)	188 (87,9%)	214 (100%)

μ μ μ μ . μ ,  
 μμ μ  
 μ μ μμ .  
 μ μ (73,4%) μ ,  
 μ μ μ μ .  
 μ μ ,  
 μ (20,1%). μ , (12,1%)  
 μ μ μ ,  
 μ μ μ ,  
 μ μ (10,3%).

4.18

	( )		
( μ μ ) μ	128 (59,8%)	86 (40,2%)	214 (100%)
( μ μ )	10 (4,7%)	204 (95,3%)	214 (100%)
μ μ μ	61 (28,5%)	153 (71,5%)	214 (100%)
μ μ μ /	109 (50,9%)	105 (49,1%)	214 (100%)
μ μ μ 180°	25 (11,7%)	189 (88,3%)	214 (100%)

μ μ μ 180°

μ μ μ (59,8%), μ / (50,9%)

μ μ μ μ (28,5%). << >>

1 20

(4,7%). (32,7%)

4.6.2

μμ

μ μ μμ

μ

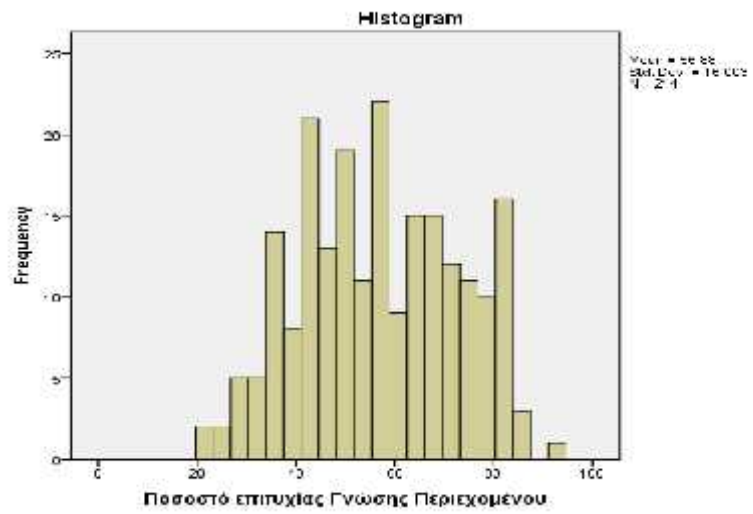
μ μ μμ

μ μ μ μ

μ μ μ μ

4.19

μ



4.19

μ

μ

μ  
56,88%.

μ 214

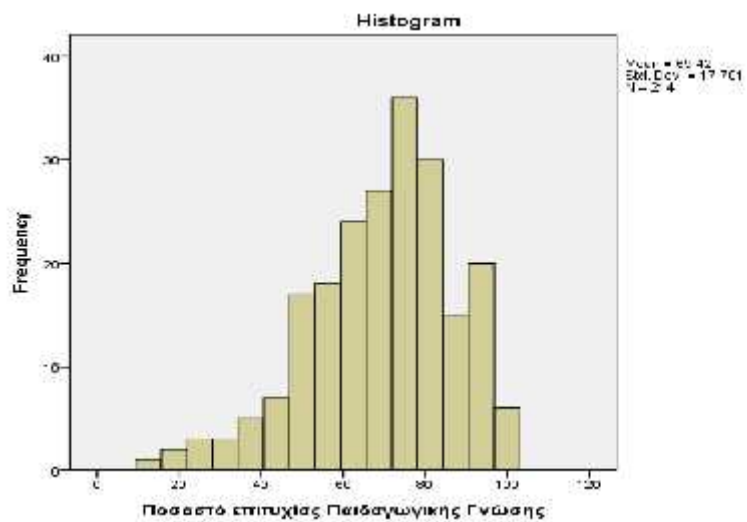
μ

20 %

85%, μ

90%.

4.20



4.20

μ

μ

μ

μ  
69,42%,

μ 214

μ

μ



4.23			μ					2d
			μ		μ		0,80	
			0,90					
				μ				
			17	26	50	1	33	127
	%	μμ	65,4%	61,9%	50,0%	50,0%	84,6%	60,8%
			9	16	50	1	6	82
	%	μμ	34,6%	38,1%	50,0%	50,0%	15,4%	39,2%
			26	42	100	2	39	209
	%	μμ	100,0%	100,0%	100,0%	100,0%	100,0%	100,0%

4.23 61,9% (26 μ )  
, μ μ  
38,1% (16 μ ) . μ  
μ (x<sup>2</sup>(4)=14,519, p=0,006<0,05).

4.24			μ					2e
			μ		μ		0,80	
			0,90					
				μ				
			41	16	41	1	28	127
	%	μμ	70,7%	66,7%	48,2%	20,0%	75,7%	60,8%
			17	8	44	4	9	82
	%	μμ	29,3%	33,3%	51,8%	80,0%	24,3%	39,2%
			58	24	85	5	37	209
	%	μμ	100,0%	100,0%	100,0%	100,0%	100,0%	100,0%

4.24 66,7% (16 μ )  
, μ μ  
33,3% (8 μ ) . μ  
μ (x<sup>2</sup>(4)=15,280, p=0,004<0,05).

4.25			μ			4b
			0,125			
			;			
			75	46	6	127
	%	μμ	52,8%	79,3%	66,7%	60,8%
			67	12	3	82
	%	μμ	47,2%	20,7%	33,3%	39,2%
			142	58	9	209
	%	μμ	100,0%	100,0%	100,0%	100,0%

4.25 52,8% (75 μ )  
, μ μ

47,2% (67 μ) . μ  
 μ (x<sup>2</sup>(2)=12,261, p=0,002<0,05).

**4.26** μ 4c

		0,125		0,125 % ;		
		30	93	4		127
	% μμ	85,7%	56,0%	50,0%		60,8%
		5	73	4		82
μ	% μμ	14,3%	44,0%	50,0%		39,2%
		35	166	8		209
	% μμ	100,0%	100,0%	100,0%		100,0%

4.26 56% (93 μ)  
 , μ μ

44% (73 μ) . μ  
 μ (x<sup>2</sup>(2)=11,092, p=0,004<0,05).

**4.27** μ 4e

		0,125		1		
		25 ;				
		54	64	9		127
	% μμ	43,2%	91,4%	64,3%		60,8%
		71	6	5		82
μ	% μμ	56,8%	8,6%	35,7%		39,2%
		125	70	14		209
	% μμ	100,0%	100,0%	100,0%		100,0%

4.27 56,8% (71 μ)  
 , μ μ

43,2% (54 μ) . μ  
 μ (x<sup>2</sup>(2)=43,856, p=0,001<0,05).

**4.28** μ 5

		μ μ ;					
		μ μ		μ			
			μμ				
		25	31	21	43	7	127
	% μμ	65,8%	79,5%	65,6%	47,3%	77,8%	60,8%
		13	8	11	48	2	82
μ	% μμ	34,2%	20,5%	34,4%	52,7%	22,2%	39,2%
		38	39	32	91	9	209
	% μμ	100,0%	100,0%	100,0%	100,0%	100,0%	100,0%



4.28 52,7% (48 μ ) μ  
 , μ μ  
 47,3% (43 μ ) μ  
 μ (x<sup>2</sup>(4)=14,515 , p=0,006<0,05).

**4.29** μ ба

			μ ;					
			21	23	24			
μ	%	μμ	22 62,9%	74 62,7%	3 33,3%	19 50,0%	9 100,0%	127 60,8%
	%	μμ	13 37,1%	44 37,3%	6 66,7%	19 50,0%	0 0,0%	82 39,2%
	%	μμ	35 100,0%	118 100,0%	9 100,0%	38 100,0%	9 100,0%	209 100,0%
	%	μμ	100,0%	100,0%	100,0%	100,0%	100,0%	100,0%

μ μ 4.29 62,7% (74 μ ) μ  
 , μ μ  
 37,3% (44 μ ) μ  
 μ (x<sup>2</sup>(4)=10,751, p=0,03<0,05).

**4.30** μ 7

			1, μ 9					
			0	0	0	2		
μ	%	μμ	96 66,2%	10 71,4%	9 75,0%	10 27,8%	2 100,0%	127 60,8%
	%	μμ	49 33,8%	4 28,6%	3 25,0%	26 72,2%	0 0,0%	82 39,2%
	%	μμ	145 100,0%	14 100,0%	12 100,0%	36 100,0%	2 100,0%	209 100,0%
	%	μμ	100,0%	100,0%	100,0%	100,0%	100,0%	100,0%

4.30 72,2% (26 μ ) μ  
 , μ μ  
 27,8% (10 μ ) μ  
 μ (x<sup>2</sup>(4)=21,211, p=0,001<0,05).

4.31			μ		8d	
			H μ 4 ;			
			101	26	127	
	%	μμ	57,1%	81,2%	60,8%	
μ			76	6	82	
	%	μμ	42,9%	18,8%	39,2%	
			177	32	209	
	%	μμ	100,0%	100,0%	100,0%	

μ μ 4.31 57,1% (101 μ )  
 , μ μ  
 42,9% (76 μ ) . μ  
 μ (x<sup>2</sup>(1)=6,650, p=0,01<0,05).

4.32			μ		8e	
			H μ 5 ;			
			86	41	127	
	%	μμ	54,8%	78,8%	60,8%	
μ			71	11	82	
	%	μμ	45,2%	21,2%	39,2%	
			157	52	209	
	%	μμ	100,0%	100,0%	100,0%	

4.32 54,8% (86 μ )  
 , μ μ  
 45,2% (71 μ ) μ . μ  
 μ (x<sup>2</sup>(1)=9,492, p=0,002<0,05).

4.33			μ			9a	
			μμ				
			86	40	1	127	
	%	μμ	53,4%	85,1%	100,0%	60,8%	
μ			75	7	0	82	
	%	μμ	46,6%	14,9%	0,0%	39,2%	
			161	47	1	209	
	%	μμ	100,0%	100,0%	100,0%	100,0%	

4.33 53,4% (86 μ )  
 , μ μ  
 46,6% (75 μ ) μ . μ  
 μ (x<sup>2</sup>(2)=15,973, p=0,001<0,05).

4.34		μ		9b	
		μμ			
	%	μμ	52	75	127
			85,2%	50,7%	60,8%
μ	%	μμ	9	73	82
			14,8%	49,3%	39,2%
	%	μμ	61	148	209
			100,0%	100,0%	100,0%

μ μ 4.34 50,7% (75 μ )  
, μ μ  
49,3% (73 μ ) . μ  
μ μ ( $\chi^2(1)=21,653, p=0,001<0,05$ ).

4.35		μ		9c	
		μ		μμ	
	%	μμ	93	33	1
			56,0%	80,5%	50,0%
μ	%	μμ	73	8	1
			44,0%	19,5%	50,0%
	%	μμ	166	41	2
			100,0%	100,0%	100,0%

4.35 56% (93 μ )  
, μ μ  
44% (73 μ ) μ . μ  
μ μ ( $\chi^2(2)=8,352, p=0,015<0,05$ ).

4.36		μ		9f	
		μ		, μμ	
	%	μμ	50	76	1
			83,3%	51,4%	100,0%
μ	%	μμ	10	72	0
			16,7%	48,6%	0,0%
	%	μμ	60	148	1
			100,0%	100,0%	100,0%

4.36 51,4% (76 μ )  
, μ μ  
48,6% (72 μ ) μ . μ  
μ μ ( $\chi^2(2)=18,965, p=0,001<0,05$ ).

**4.37**  $\mu$   $\mu$   $\mu$  (  $\mu$  )

			$\mu$ %		$\mu$ $\mu$
$\mu$ $\mu$ ( $\mu$ )	$\mu$	127	33,9	32,661	2,898
		82	51,2	38,470	4,248

$\mu$   $\mu$   $\mu$  4.37  $\mu$   
 $\mu$   $\mu$   $\mu$  ,  
 $\mu$   $\mu$   $\mu$   $\mu$  ,  
 (51,2% 33,9% ),  $t(207)=-3,497$ ,  $p=0,001<0,05$

**4.38**  $\mu$  (  $\mu$  )  $\mu$

			$\mu$ %		$\mu$ $\mu$
$\mu$ ( $\mu$ )	$\mu$	127	42,9	18,228	1,617
		82	58,6	18,631	2,057

4.38  
 $\mu$   $\mu$   $\mu$  ,  $\mu$   
 $\mu$   $\mu$   $\mu$  , (58,6%  
 42,9% ),  $t(207)=-6,012$ ,  $p=0,001<0,05$

**4.39**  $\mu$  (  $\mu$  )  $\mu$

			$\mu$ %		$\mu$ $\mu$
$\mu$ ( $\mu$ )	$\mu$	127	64,2	18,777	1,666
		82	76,6	15,500	1,712

4.39  $\mu$   
 $\mu$   $\mu$   $\mu$  ,  $\mu$   
 $\mu$   $\mu$   $\mu$  , (76,6%  
 64,2% ),  $t(207)=-4,998$ ,  $p=0,001<0,05$

**4.40** (  $\mu$  )  $\mu$

			$\mu$ %		$\mu$ $\mu$
( $\mu$ )	$\mu$	127	7,9	27,040	2,399
		82	31,7	46,820	5,170

4.40  $\mu$   $\mu$  ,  $\mu$  (31,7%  
 $\mu$   $\mu$  ,  $\mu$  ,  
 7,9% ),  $t(207)=-4,661, p=0,001<0,05$

4.6.4  $\mu$   $\mu$   $\mu$   $\mu$   $\mu$   $\mu$  .  
 $\mu$   $\mu$  .

**4.41.**  $\mu$  C1e

		0		
		$\mu$		
		35	92	127
	% $\mu\mu$	77,8%	56,1%	60,8%
$\mu$		10	72	82
	% $\mu\mu$	22,2%	43,9%	39,2%
		45	164	209
	% $\mu\mu$	100,0%	100,0%	100,0%

4.41 77,8% (35  $\mu$  )  
 ,  $\mu$   $\mu$   $\mu$   
 22,2% (45  $\mu$  )  $\mu$   $\mu$   $\mu$   
 $\mu$   $\mu$  ( $\chi^2(1)=6,962, p=0,008<0,05$ ).

**4.42**  $\mu$  C3a

		$\mu$ 1		
		11	116	127
	% $\mu\mu$	35,5%	65,2%	60,8%
$\mu$		20	62	82
	% $\mu\mu$	64,5%	34,8%	39,2%
		31	178	209
	% $\mu\mu$	100,0%	100,0%	100,0%

$\mu$   $\mu$  4.42 64,5% (20  $\mu$  )  $\mu$   
 ,  $\mu$   $\mu$   $\mu$   
 35,5% (11  $\mu$  )  $\mu$   $\mu$   $\mu$   
 $\mu$   $\mu$  ( $\chi^2(1)=9,758, p=0,002<0,05$ ).

4.43			$\mu$		C4c
			$\mu$		
	%	$\mu\mu$	56	71	127
			47,9%	77,2%	60,8%
$\mu$	%	$\mu\mu$	61	21	82
			52,1%	22,8%	39,2%
			117	92	209
	%	$\mu\mu$	100,0%	100,0%	100,0%

4.43 52,1% (61  $\mu$ )  
,  $\mu$   $\mu$   
47,9% (56  $\mu$ ) .  $\mu$   
 $\mu$  ( $\chi^2(1)=18,559, p=0,001<0,05$ ).

4.44			$\mu$		C6d
			$\mu$		
	%	$\mu\mu$	95	32	127
			65,1%	50,8%	60,8%
$\mu$	%	$\mu\mu$	51	31	82
			34,9%	49,2%	39,2%
			146	63	209
	%	$\mu\mu$	100,0%	100,0%	100,0%

4.44 65,1% (95  $\mu$ )  
,  $\mu$   $\mu$   
34,9% (51  $\mu$ ) .  $\mu$   
 $\mu$  ( $\chi^2(1)=3,762, p=0,049<0,05$ ).

4.45			$\mu$		C7b
			$\mu$		
	%	$\mu\mu$	86	41	127
			56,2%	73,2%	60,8%
$\mu$	%	$\mu\mu$	67	15	82
			43,8%	26,8%	39,2%
			153	56	209
	%	$\mu\mu$	100,0%	100,0%	100,0%

4.45 56,2% (86  $\mu$ )  
,  $\mu$   $\mu$   
43,8% (67  $\mu$ ) .  $\mu$   
 $\mu$  ( $\chi^2(1)=4,972, p=0,026<0,05$ ).

**4.46**

μ

C8a

		μ ( μ )		
		μ	μ	
	% μμ	66 52,4%	61 73,5%	127 60,8%
μ	% μμ	60 47,6%	22 26,5%	82 39,2%
	% μμ	126 100,0%	83 100,0%	209 100,0%

4.46 52,4% (66 μ )

, μ μ

47,6% (60 μ )

. μ

μ (x<sup>2</sup>(1)=9,356, p=0,002<0,05).

**4.47**

μ

μ

( )

			μ %		μ μ
( μ )	μ	127	71	22,243	1,974
		82	79,1	23,102	2,551

μ μ 4.47 μ

μ μ , μ

μ , (79,1%

71% ), t(207)=-2,522, p=0,012<0,05

**4.48.**

μ

μ

( )

			μ %		μ μ
( μ )	μ	127	68,6	22,651	2,010
		82	77,9	22,248	2,457

4.48,

μ μ , μ

μ , (77,9%

68,6% ), t(207)=-2,917, p=0,004<0,05

**4.49**

μ ( )					
			μ %		μ μ
( μ )	μ	127	41,7	34,797	3,088
		82	61,9	34,522	3,812

μ μ 4.49 μ  
 μ μ , μ  
 μ , (61,9% 41,7%)  
 ), t(207)=-4,102, p=0,001<0,05

**4.50**

μ ( ) μ					
			μ %		μ μ
( μ )	μ	127	69,5	27,443	2,435
		82	78,6	20,699	2,286

4.50  
 μ μ μ , μ  
 , (78,6% 69,5%)  
 ), t(207)=-2,501, p=0,013<0,05

**4.51**

μ μ					
			μ %		μ μ
		127	50,1	14,484	1,285
μ	μ	82	65,5	14,332	1,583
	μ	127	65,9	16,737	1,485
		82	74,8	17,933	1,980

μ μ , 4.51 μ  
 μ μ ,  
 μ [65,5% 50,1%]  
 , t(207)=-7,133, p=0,01<0,05]  
 [74,8% 65,9% , t(207)=-3,660, p=0,01<0,05]

**4.6.5**

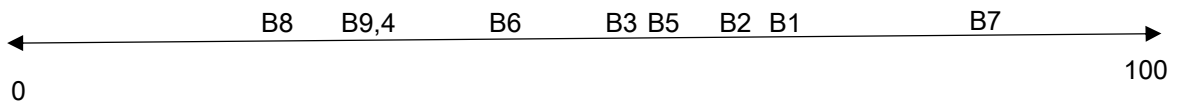
-

μ μ μ μ μ μ μ μ



4.52 μ ( )

		(%)
1	μ μ	65,9%
2	μ	60,6%
3	μ μ	52,3%
4	μ	30,5%
5	μ	56,5%
6	μ	43,5%
7		82,7%
8	μ	25,2%
9	μ	30%



4.52 μ , , μ , μ  
 . μ ,  
 μ (82,7%).  
 μ μ μ  
 μ (65,9% 60,6% ).  
 μ μ  
 μ (56,5% 52,3% ).  
 μ μ (25,2%).

**4.53**

μ ( )

		(% )
1	μ μ	70,8%
2	μ	68,7%
3	μ μ	61,4%
4	μ	41,1%
5	μ	66,1%
6	μ	41,7%
7		92,1%
8	μ	28,7%
9	μ	36,6%

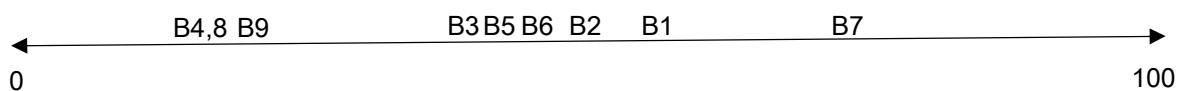


4.53 μ ,  
 , μ ,  
 μ .  
 μ ,  
 μ (92,1%). μ  
 μ , μ  
 μ μ (70,8%, 68,7% 66,1% ).  
 μ (61,4%).  
 μ μ  
 (28,7%).

**4.54**

μ ( μ )

		(% )
1	μ μ	58,5%
2	μ	49,7%
3	μ μ	39%
4	μ	20,2%
5	μ	41,4%
6	μ	46,3%
7		68,3%
8	μ	20,5%
9	μ	22,8%

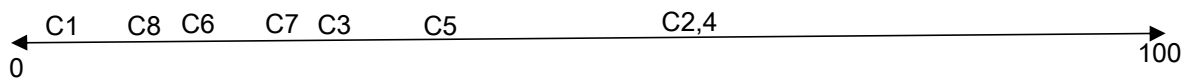




μ (31,8%).  
 μ ( μ )  
 μ ( μ )  
 μ 8,2%.

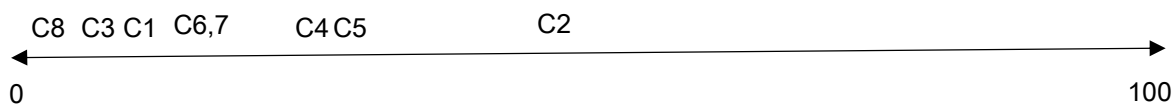
**4.56** ( )

		(% )
C1	( μ )	7,9%
C2	μ ( )	55,1%
C3	μ ( μ )	28,3%
C4	μ ( μ )	56%
C5	( μ )	35,4%
C6	( μ )	14,2%
C7	( μ )	26%
C8	μ ( μ )	11%



4.56 μ ,  
 ,  
 μ .  
 , μ  
 μ (56% 55,1% ). μ μ  
 μ (35,4%).  
 μ  
 μ (7,9%).

4.57		
	( μ )	(% )
C1	( μ )	9,8%
C2	μ ( )	45,1%
C3	μ ( μ )	6%
C4	μ ( μ )	25,6%
C5	( μ )	28%
C6	( μ )	14,6%
C7	( μ )	15,6%
C8	μ ( μ )	4,9%



4.57 μ , , μ , μ (45,1%). μ (28%). μ μ (4,9%).





5

μ μ

μ

μ μ

(μ μ )

μ

μ

.

μ

: )

, )

μ μ

μ )

μ

μ ,

μ

μ

μ

μ

μ

μ

,

μ

μ

μ

μ

μ

.

μ

,

μ

(

82,7%),

μ

.

μ

Turnuklu & Yesildere (2007)

μ

.

,

μ

μ

μ

μ

μμ

(

65,9%).

μ

Olfos, Goldrine & Estrella (2014) μ

μ

,

μ

μ

(37%

57%

).

μ

Turnuklu & Yesildere (2007). μ

μ

μ

μ

(

60,6%)

μ

.

μ

μ

μ

(μ )

μ

μ

(

43,5%)

μ

μ

μ

.

μ

μ

Maniraho & Christiansen (2015)





μ μ μ  
 μ μ .  
 μ μ  
 Ward & Thomas (2006) 64% μ  
 μ μ  
 μ . μ 33%  
 μ μ μ . μ ,  
 (2007) μ μ μ μ μ μ Turnuklu & Yesildere  
 μ μ μ μ .  
 ,  
 ,  
 μ μ Olfos, Goldrine &  
 Estrella (2014)  
 50%.  
 μ μμ  
 μ μ μ μ .  
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- Andelginger, B. (1981). Provocative texts and spontaneous reactions of teachers: A method for recognizing teaching and learning of mathematics. Equipe de Recherche Pédagogique (Eds.), *Proceedings of the 5th PME International Conference*, Vo 1, ( . 381- 386). Singapore:
- Ball, D. L., Thames, M. H., Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), ( . 389-407)
- Ball, D. L., Thames M. H., Bass, H., Sleep, L., Lewis, J., Phelps, G. (2009a). A practicebased theory of mathematical knowledge for teaching. M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of the 33th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, ( . 95-98). Thessaloniki:PME
- Ball, D.L., Charalambous, C.Y., Thames, M., & Lewis, J.M. (2009). Teacher knowledge and teaching: viewing a complex relationship from three perspectives. M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, Vol 1 ( . 121-125). Thessaloniki:AUTH
- Batthey, D., Franke, M. L., Kazemi, E. (2007). *Mathematics teaching and classroom practice*. Greenwich: Information Age Publishers
- Battista, M. (1999). *The Mathematical Miseducation of America's Youth*. Phi Delta Kappan
- Bavaria, R., & Sabeen, M. (2005). *Sylvan Learning Center Math Research*. Sylvan Learning
- Beswick, K., Goos, M., (2012). Measuring Pre-service Primary Teachers' Knowledge for Teaching Mathematics. *Mathematics Teacher Education and Development*, Vol. 14.2, ( . 70–90)
- Bright, G. W., Bowman, A., Vacc, N. (1997). Teachers' frameworks for understanding children's mathematical thinking. H. Pekkonen (Ed.), *Proceedings of the 21st PME International Conference*, ( . 84-90). Lanti, Finland:PME
- Callingham, R. (2011). Beginning teachers' mathematical knowledge: What is needed?. *Australian Association of Mathematics Teachers (AAMT) and the Mathematics Education Research Group of Australasia*, ( . 828-835). Australia:AAMT and ERGA

- Chapman, O. (2004). Facilitating peer interactions in learning mathematics: Teachers' practical knowledge. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Vol 2, ( . 191–198). Bergen, Norway:PME
- Chapman, O., & Ponte, J. (2006). Mathematics teachers' knowledge and practices. A. Gutierrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future*, ( . 461-494). Rotterdam: Sense
- Chick, H. Baker, M. Pham, T. Cheng, H. (2006). Aspects of Teachers' Pedagogical Content Knowledge for Decimals, *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*, Vol 2, ( .297-304). Prague, Czech Republic: PME
- Chick, H. Baker, . (2005). Investigating teacher's responses to student misconception. *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*. Vol 2, ( . 255-262). Melbourne, Australia:PME
- Chick, H. Pierce, R. (2008). Teaching statistics at the primary school level: beliefs, affordance and pedagogical content knowledge. C. Batanero, G. Burrill, C. Reading & A. Rossman (Eds.), *Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*. Paris. France:ICMI
- Cuba, E. G. & Lincoln, Y.S., (1994). Competing paradigms in qualitative research. N.L. Denzin & Y.S.Lincoln (Eds.), *Handbook of qualitative research*, ( .105-117). CA: Sage
- D'Ambrosio, U. (1995). Ethnomathematic: theory and pedagogical practice (second part). *L Educazione Matematica*, v.17, n.01, ( . 29-48)
- Even, R., & Markovits, Z., (1991). Teachers' pedagogical knowledge: The case of functions. F. Furinghetti (Ed.), *Proceedings of the 15th PME International Conference*, 2, ( . 40-47). Assisi:PME
- Even, R., (2009). Teacher knowledge and teaching: Considering the connections between perspectives and findings. M. Tzekaki, M. Kaldrimidou, & H. Sakonidis, (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, 1, ( . 147-148)
- Freudenthal, H. (1968). *The legacy of Hans Freudenthal*. Ulrecht. Kluwer Academic Publisher



- Gal, H., & Vinner, S., (1997). Perpendicular lines: What is the problem? Pre-service Teachers lack of knowledge on how to cope with students' difficulties. H. Pekkonen (Ed.), *Proceedings of the 21st PME International Conference*, 2 ( . 281-288). Lanti, Finland:PME
- Freire, . (1977).  $\mu$  . . . : -
- Hashweh, M. (1987). Effects of subject-matter knowledge in the teaching of biology and physics. *Teaching and Teacher Education*, 3(2), ( . 109–120)
- Hewson, P., W. & Hewson, M. G., (1989). Analysis and use of a task for identifying conceptions of teaching science. *Journal of Education for Teaching*, 15(3), ( . 191–208)
- Johnson, J. (2000). *Teaching and Learning Mathematics*. Olympia WA: Office of Superintendent of Public Instruction
- Kambilombilo, D., & Sakala,W. (2015). An Investigation into the Challenges In-Service Student Teachers Encounter in Transformational Geometry “Reflection and Rotation”. *The Case of Mufulira College of Education. Journal of Education and Practice*. Vol.6, No2, ( . 139-149)
- Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser M., Krauss, S. & Baumert, J., (2013). Teachers' Content Knowledge and Pedagogical Content Knowledge: The Role of Structural Differences in Teacher Education. *Journal of Teacher Education*, 64(1), ( . 90–106)
- Klein, R., & Tirosh, D., (1997). Teachers' pedagogical content knowledge of multiplication and division of rational numbers. H. Pekkonen (Ed.), *Proceedings of the 21st PME International Conference*, 3, ( . 144-152). Lanti, Finland:PME
- Magiera, M., Leigh, A., & Moyer, J., (2013). *An exploratory study of pre-service middle school teachers' knowledge of algebraic thinking*. Educational Studies in Mathematics, Vol. 84, No. 1, ( . 93-113), New York, Springer
- Maniraho, J., Christiansen, I. (2015). Rwandan grade 6 mathematics teachers' knowledge. *Rwandan Journal of Education*, Vol 3, No. 1, ( . 148-158)
- McCray, J. (2008). *Pedagogical content knowledge for preschool mathematics: Relationships to teaching practices and child outcomes*. Loyola University Chicago: Erikson Institute
- National Council of Teachers of Mathematics*. 1 2017  
The Use of Technology in Learning and Teaching of Mathematics:  
[http://nctm.org/about/position\\_statements/position\\_statement\\_13.htm](http://nctm.org/about/position_statements/position_statement_13.htm)
- National Staff Development Council. (2005). *A Study of Professional Development for Public School Educators in West Virginia*. Oxford

- eubrand, M., & Seago, N., (2008). The balance of teacher knowledge: Mathematics and pedagogy. R. Even & D.L. Ball (Eds.), *The professional education and development of teachers of mathematics* ( . 211-225). Berlin, Heidelberg. New York: Springer
- Nunes, T., Carraher, D., & Schielmann, D. (1994). *Street mathematics and school mathematics*. Cambridge: Cambridge University Press
- Olfos, R., Estrella, S. (2010). Chilean Primary Teachers Challenged to build PCK for Statistics. Instituto de Matemática, *Pontifical Catholic University de Valparaíso*, Chile. Center for Advanced Research in Education. University of Chile
- R Olfos, ., Goldrine, ., Estrella, S. (2014). Teachers' pedagogical content knowledge and its relation with students' understanding. *Revista Brasileira de Educação*, v. 19, n. 59. Rio de Janeiro
- Papert, S., & Harel, L. (1991). *Constructionism*. Westport. CT: Ablex Publishing
- Ponte, J. (1994). Mathematics teachers' professional knowledge (plenary conference). J. P. Ponte & J. F. Matos (Orgs.), *Proceedings of the XVIII International Conference for the Psychology of Mathematics Education*. Vol. I, ( . 195-210). Lisbon, Portugal:PME.
- Protheroe, N. (2007). *What Does Good Math Instruction Look Like?*. *Principal*, v87, n1, ( . 51-54)
- Rees, R., (1982). The teacher and diagnosis: Too much Piaget. A. Vermandel (Ed.), *Proceedings of the 6th PME International Conference*, 2 ( . 91-96), Antwerp, Belgium:PME
- Roche, A., & Clarke, D. (2011). Some lessons learned from the experience of assessing teacher pedagogical content knowledge in mathematics. Mathematics: Traditions and new practices, *Proceedings of the AAMT-MERGA conference held in Alice Springs*
- Roche, A., & Clarke, D. (2009). Making Sense of Partitive and Quotitive Division: A Snapshot of Teachers' Pedagogical Content Knowledge. R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia*, (Vol. 2). Palmerston North, NZ: MERGA.
- Rossouw, L., & Smith, E., (1998). Teachers' pedagogical content knowledge of geometry. A. Olivier & K.Newstead (Eds.), *Proceedings of the 22nd PME International Conference*, 4 ( . 57-63). Tucson, Arizona:PME
- Romberg, T. (2000). *Changing the teaching and learning of mathematics*. v56, n4,( . 6-9). Australian Mathematics Teacher

- Schifter, D., & Simon, M. A. (1991). *Toward a constructivist perspective: An intervention study of mathematics teacher development*. Educational Studies in Mathematics
- Shulman, L., S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), ( .1-23)
- Silverman, J., & Thompson, P., (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education*, 77(6), ( . 499-511)
- Smith, D. C. & Neale, D. C. (1991). The construction of subject-matter knowledge in primary science teaching. *J. Brophy (Eds.), Advances in Research on Teaching*, vol 2, ( . 1-9)
- Teaching Today (2005a)*. 31 2017 : Standards-Based Instruction in Mathematics: <http://www.glencoe.com/sec/teachingtoday/subjects>
- Teaching Today (2005b)*. 31 2017 : Meeting Middle School Math Standards: <http://www.glencoe.com/sec/teachingtoday/subject/meeting-msstandards..phtml>
- Turnuklu, E., Yesildere, S., (2007). The Pedagogical Content Knowledge in Mathematics: Pre-Service Primary Mathematics Teachers' Perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, vol 1. Turkey
- Ward, G., Thomas, G. (2006). *Findings from the New Zealand Numeracy Development Projects*. New Zealand: Minister of Education
- Wiegel, H., & Steffe, L. (1992). *On reforming practice in mathematics education* *Educational Studies in Mathematics*. Volume 23, Issue 5, ( . 445–465). International Journal of Development Research
- Yeo, J. (2008). Teaching area and perimeter: Mathematics-pedagogical-content knowledge-in-action. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia*. Brisbane,Australia:MERGA



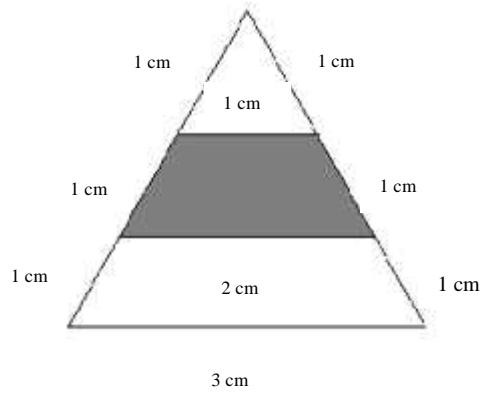
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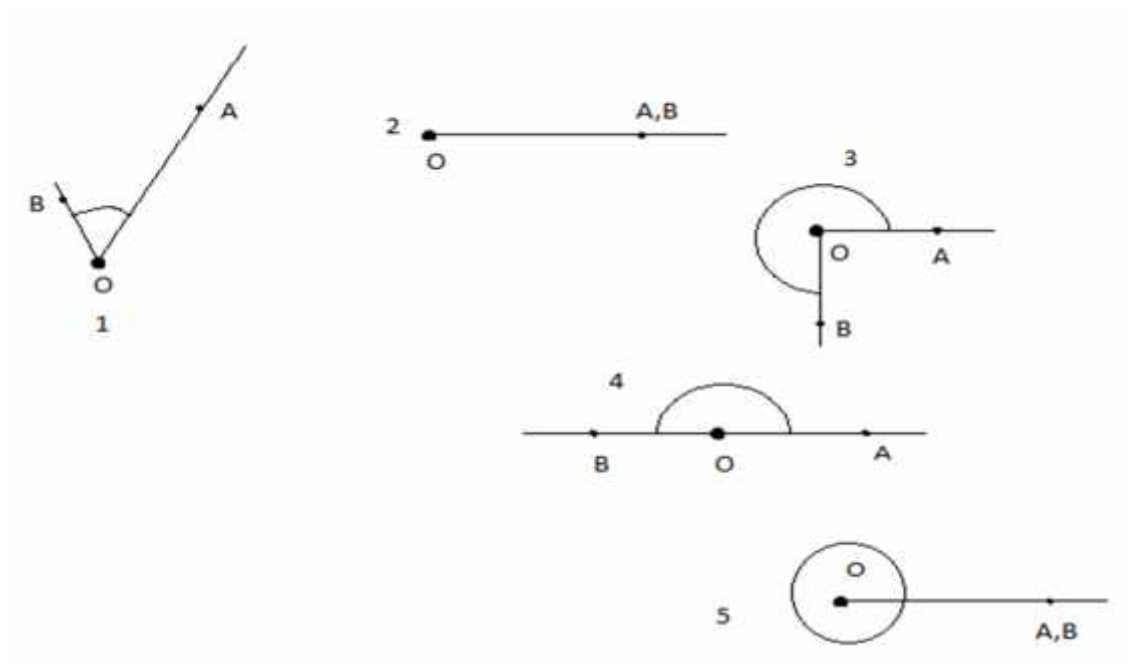
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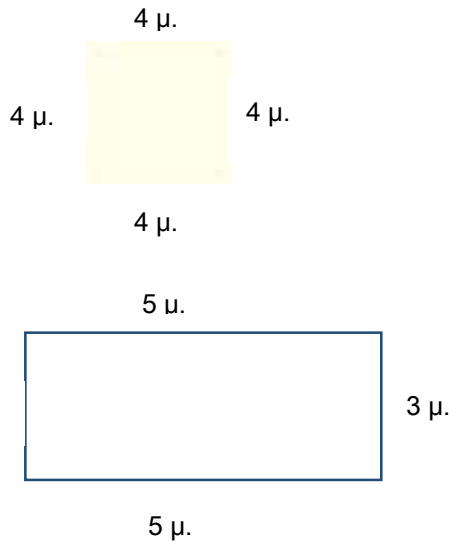






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