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Abstract..... 3

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2.1. μ μ ..... 5

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2.2. μ μ ..... 10

2.3. μ μ ..... 11

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2.4. .... 13

3. .... 14

3.1. μμ ..... 14

3.2. μ ..... 15

3.3. .... 18

3.4. μ ..... 19

4. μ ..... 20

5. .... 38

6. .... 41

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## **Abstract**

Rational numbers are one of the most significant topics in the teaching of Mathematics in Primary Education in which students and teachers find difficulties. Teacher should have appropriate Content Knowledge (CK) and Pedagogical Content Knowledge (PCK) to be able to teach their students effectively. However, few are aware about prospective teacher's Content Knowledge (CK) and Pedagogical Content Knowledge (PCK) on rational numbers in Greece. therefore, the aim of this study was to investigate prospective teacher's CK and PCK on rational numbers as well as their correlation. Semi- structured interviews were taken from 45 primary teachers for the collection of the data of current study. The results showed that prospective teachers have difficulties in rational numbers and CK and PCK are limited in some prospective teacher's who present significant learning gaps. Additionally, is revealed a positive correlation between teacher candidates' CK and PCK.

**Keywords:** Rational numbers, Content Knowledge (CK), Pedagogical Content Knowledge (PCK), prospective teachers











COACTIV (Professional Competence of Teachers, Cognitively  
Activating Instruction, and the Development of Students' Mathematical Literacy)

2003-2004, (Grade 10)  
(Baumert et al., 2010).

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COACTIV μ

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COACTIV μ

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Kleickmann et al., (2013) COACTIV μ





Dooren, & Verschaffel, 2012). (Vamvakoussi, Van  
 $2/5 + 3/4 = 5/9$ ,  $2+3$   $\mu$  5  $5+4 \mu$  9 (Depaepe et al.,  
 2014).

(Kieren, 1993; Moskal & Magone, 2000; Lamon, 1999; Stafylidou & Vosniadou, 2004).  
 O Stafylidou & Vosniadou, (2004)

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 (1991)  
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**2.3.**

Turnuklu & Yelsidere (2007)  
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Depaere et al., (2015)

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Tirosh (2000)

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μ μ μ , μ :

- : : 3/4-1/4
- : : 3:0,5
- : : 4x3/4
- : μ μ 3/4 μ μ μμ
- : μ 1/8 μ . μμ ;  
 3/4 μ μ 1/3 ;

, 13 μ μ

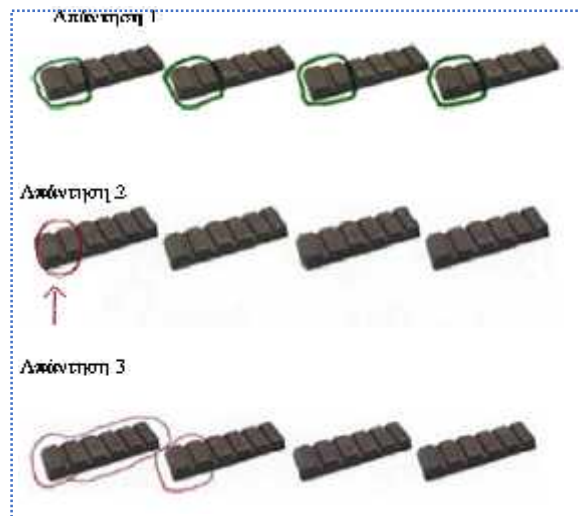
:

- : μ , μ ; 3/8 & 7/8
- : μ , μ ; 2/4 & 4/2
- : μ , μ ; 1/2 & 5/8
- : μ , μ ; 3/7 & 5/8
- 1: μ 9/8 μ μ
- μ 4;
- 2: ;
- 1: μ 3/4 μ μμ .
- 2: μ 3/8 μ μμ .
- 1: 150% 12.
- 2: 90% 40.
- : μ μ 7/8 1.
- : 1/4 5 .
- : μ μ μ μ : 0,33 0,8  
 0,242 0,4 0,71



$\mu$   $\mu$   $\mu$   $\mu$  /  $\mu$   
 $\mu$   $\mu$   $\mu$  /  $\mu$  (1, 2, 3,  
 ..7, 8, ..)  
 ΕΑΚ,  $\mu$   
 «  $\mu$  1/3 »

1:  $\mu$



$\mu$   $\mu$   $\mu$   $\mu$   
 $\mu$   $\mu$   $\mu$   $\mu$  1  $\mu$   $\mu$   
 $\mu$   $\mu$   $\mu$   $\mu$  (6  $\mu$  )  
 $\mu$   $\mu$   $\mu$   $\mu$  2  $\mu$  (1/3).  $\mu$  2,  $\mu$   $\mu$   
 $\mu$   $\mu$   $\mu$   $\mu$  3,  $\mu$   $\mu$  4 (  $\mu$  24  
 $\mu$   $\mu$  ) 1/3,  $\mu$   
 ( 6  $\mu$  ) 2  $\mu$   $\mu$  .  
 $\mu$   $\mu$   $\mu$   $\mu$   $\mu$   $\mu$   $\mu$   $\mu$   $\mu$  : 0,33 0,8  
 0,242 0,4 0,71.  $\mu$   $\mu$   $\mu$   $\mu$  :  
 0.3 0.4 0.72 0.53 0,475

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 « μ μ 3/4 μ . μμ  
 μ 1/8 μ . μμ ;»  
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 μ μ μ .  
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 (2003). O 4 μ ( , , )  
 Clarke, & Roche, (2009). 2 μ  
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 Depaepe et al. (2015). , 2  
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 (2014)

<http://mathslife.eled.uowm.gr/?q=%CE%B4%CE%B9%CE%B1%CE%B3%CF%89%CE%BD%CE%B9%CF%83%CE%BC%CF%8C%CF%82-2014> ).

### 3.3.

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4. μ

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2:

ΕΡΩΤΗΣΕΙΣ ΓΠ	ΠΟΣΟΣΤΑ ΣΩΣΤΩΝ ΑΠΑΝΤΗΣΕΩΝ N=45
ΠΚΑ: 3/4 – 1/4	44 (97,8%)
ΠΔΔ: 3: 0,5	26 (57,8%)
ΠΚΠ: 4 X 3/4	37 (82,2 %)
ΣΚΟ: 3/8, 7/8	37 (82,2%)
ΣΚΑ: 2/4, 4/2	30 (66,7%)
ΣΚΜ: 1/2, 5/8	34 (75,6 %)
ΣΚΕ: 3/7, 5/8	30 (66,7%)
ΜΚ1: 9/8 ΑΝ Ο ΑΡΙΘΜΗΤΗΣ ΤΡΙΠΛΑΣΙΑΣΤΕΙ ΚΑΙ Ο ΠΑΡΟΝΟΜΑΣΤΗΣ ΥΠΟΤΕΤΡΑΠΛΑΣΙΑΣΤΕΙ	25 (55,6%)
ΜΚ2: ΑΝ ΘΑ ΜΙΚΡΑΙΝΕΙ Η ΘΑ ΜΕΓΑΛΩΣΕΙ, ΠΟΙΕΣ ΦΟΡΕΣ	9 (20%)
ΑΚ1: 3/4	36 (80,0 %)
ΑΚ2: 3/8	35 (77,8 %)
ΠΠ1: 150% ΤΟΥ 12	32 (71,1 %)
ΠΠ2: 90% ΤΟΥ 40	30 (66,7 %)
ΛΠΚ: ΚΛΑΣΜΑ ΜΕΤΑΞΥ 7/8 ΚΑΙ 1	21 (46,7%)
ΛΑΚ :1/4 ΤΩΝ 5 ΚΥΚΛΩΝ	31 (68,9%)
ΛΤΔ: 0,33 0,8 0,242 0,4 0,71	32 (71,1%)
ΛΠΔΚ: 3/4: 1/8	35 (77,8%)
ΛΠΠΚ: 3/4 X 1/3	23 (51,1%)



$\mu$   $\mu$   $\mu$   $\mu$  (3: 0,5) ,  
 $\mu$   $\mu$   $\mu$  (4 x3/4).  
**4:**  $\mu$   $\mu$

	<b>: 3/4 -1/4</b>	<b>: 3: 0,5</b>	<b>: 4x 3/4</b>
	44 (97,8%)	26 (57,8%)	37 (82,2 %)
	1 (2,2%)	4 (8,9%)	1 (2,2 %)

$\mu$   $\mu$   $\mu$   $\mu$  4,  $\mu$   
 $\mu$   
 $\mu$  (97,8%)  $\mu$  44  $\mu$   
 $\mu$  (82,2%)  $\mu$  ,  $\mu$   
 $\mu$  (3:0,5)  $\mu$   
 (57,8%).  
 7 (15,6%) ,  $\mu$   
 $\mu$   $\mu$  ,  $\mu$  4 4/4 4/1. ,  $\mu$   
 :  $\mu$  3 1,5  
 (17,8%),  $\mu$  1,5+1,5=3 (4,4%)  
 30:5=0,6 (11,1%).

**4.1:**  $\mu$

<b>: 3/4 -1/4</b>	$\mu$ ( $\mu$ $\mu$ $\mu$ )	97,8%
<b>: 3:0,5</b>	$\mu$ (30:1/2=30x 2=6)	15,6%
	$\mu$ ( $\mu$ 10, 30:5=6)	13,3%
	0,5 3 ( $\mu$ / )	28,9%
<b>: 4 x3/4</b>	$\mu$ (4/1x 3/4= 12/4)	33,3%
	$\mu$ ( (4x3)/4=12/4)	48,8%

$\mu$   
 ,  $\mu$   $\mu$   $\mu$   $\mu$  (3/4-1/4)  $\mu$   
 $\mu$   $\mu$  (4x3/4)  $\mu$  97,8% 48,8%  
 $\mu$   $\mu$  .



μ μ (3:0,5) (44,5%)  
 μ μ - μ  
 μ , μμ  
 μ , : μ μ μ (3/8  
 & 7/8), μ (2/4 & 4/2),  
 μ μ μ (1/2 & 5/8) , ,  
 μ μ (3/7 & 5/8).

5: μ μ

	ΣΚΟ: 3/8 & 7/8	ΣΚΑ: 2/4 & 4/2	ΣΚΜ: 1/2 & 5/8	ΣΚΕ: 3/7 & 5/8
<b>Επιτυχία</b>	37 (82,2%)	30 (66,7%)	34 (75,6 %)	30 (66,7%)
<b>Μη απάντηση</b>	0 (0,0 %)	0 (0,0%)	0 (0,0 %)	2 (4,4%)

μ , μ μ μ 5,  
 μ . μ  
 (82,2%). , μ μ , 37  
 34 μ (75,6%)  
 μ μ (66,7%)  
 μμ .  
 μ ( )  
 10 (22,2%) μ  
 (4/2). « 2/4 μ  
 μ » (8,9%) « 2/4 μ  
 (2,2%) (11,1%). , 9 μμ  
 (20,0%) , 5  
 5/8 μ ».  
 « μ μ μ μ »  
 , (13,3%) .



μ μ 4 ( ). μ ,  
 μ μ , μ μ  
 , 2, μ μ , .

6: μ μ μ

	1: μ 9/8 μ ;	2: μ ;
	25 (55,6%)	9 (20%)
	1 (2,2%)	19 (42,2%)

μ μ μ 6,

2. μ 25 (55,6%) μ 9  
 μ μ (33,3%) μ .  
 μ μ μ μ (20,0%) μ (13,0%).  
 μ μ μ μ ( 2), 6 (13,3%)  
 13 (24,4%)  
 μ .

6.1: μ μ

	μ μ : μ μ μ	20,0%
	μ μ μ μ	
1: μ 9/8 μ μ ;	μ ( μ ) (27/2=13,5 9/8≈1,1)	15,6%
	μ 1 ( 9/8 μ 1 27/2 μ )	13,3%
	μ μ μ μ	2,2%
2: μ μ ;	μ 1,1 13,5 (9/8=1,1 27/2= 13,5) , 12	11,1%
	μ μ μ 3 μ μ 4 μ (3x4 =12)	4,4%



7.1:

μ μ μ μμ

1: 3/4	μ μ μ μ 1/2 1, 33,3%
	1 ( μ μ μ ) 4 μμ (4/4) μ μμ (3/4)
	3/4 μ 0,75 11,1%
2: 3/8	1/2=4/8 35,6%
	1 ( μ μ μ ) 8 μμ (3/8) 28,9%

μ μ μ 7.1, 1, 15 (33,3%)  
 3/4 μ 1/2 μ 2/4 1 μ 4/4  
 14 (31,1%) μ 4 μμ μ  
 3 . , 5 (11,1%) μ , 1, μ μ  
 μ . 2, μμ (35,6%) 3/8  
 8 μμ μ 4/8 13 μμ μ  
 , μ μ  
 μ . : 1,  
 150% 12 2, 90%  
 40.

8:

μ

	<b>1: 150% 12</b>	<b>2 : 90% 40</b>
	32 (71,1 %)	30 (66,7 %)
	6 (13,3%)	6 (13,3 %)

, μ μ 7,  
 μ , 1, 32  
 (71,1%) μ 150% 12 . ,  
 2, μ 30 (66,7%).

μ , 2,9 μ (20,0%) μ 1,7 μ (15,6%) .

**8.1:**

1: 150% 12	100% 12 12 50% 12 6, 12=6=18	42,2%
	μ 150/100 x12	24,4%
	μ μ	4,4%
2: 90% 40	μ 90/100 x40	33,3%
	100% 40 40 10% 40 4, 40-4=36	15,6%
	μ μ	8,9%

μ 8.1, 19 (42,2%),  
1, μ , 100% 12 50%  
12, μ 150% 12 ( μ  
μ μ ). μ 11 μ (24,4%)  
μ μ μ , 150/100 x 12 2  
(4,4%) μ μ ( μ ).  
μ μ , 2, 90% 40  
(33,3%) μ μ 90/100 x 40 ,  
7 (15,6%), 100% 40 10% 40 (40-  
4=36). , 2, 4 (8,9%) μ μ  
.  
μ , μ μ μ  
μ μ μ μ  
μ μ : ( μ μ ) μ μ  
μ μ μ 7/8 1 ( μ  
μ μ ),  
7/8 μ μ 7/8 8/8 μ μ : « μ μ  
μ 1».

9: μ μ

	: μ μ 7/8 1	: « μ μ 7/8 1 μ μ 7/8 8/8 μ 1»
	21 (46,7%)	34 (75,6%)
	0 (0,0%)	0 (0,0%)

, μ , μ 21 (46,7%) μ μ μ 7/8 1 μ . , μ μ , μ , 34 μμ (75,6%) μ μ μ (31,1%), μ μ 7/8 1, 10 (22,2%) μ μ 8, μ , μ 11 14 ( ), μ μ .

9.1: μ μ

: μ μ 7/8 1	μμ , 8/9 9/10	26,7%
	11/12 μμ , (7x2)/(8x2)=14/16, 15/16	17,8%
	μ	μ
: « μ μ 7/8 1 μ μ 7/8 8/8 μ 1»	μ , μ μ ( / μ )	24,4%
	7 μ , μ μ μ (7/8 8/8)	22,2%
	μ , μ μ - μ μ μ	8,9%

	μ , μ	4,4%
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μ μ 7/8 1, 12 μμ (26,7%)  
μ 8/9 9/10 μ 11/12. , 8  
μ μ μ , μ μ ,  
14/15 (17,8%). μ μ μ , μ μ  
, : μ  
μ μ (24,4%)  
μ μ 7 8 μ 7/8  
8/8 μ 1 (22,2%). μ  
: « 7 8, 7/8 μ 8/8. μ  
μ μ μ μ » , «.. μ μ μ  
μμ μ 7 8 » , «.. μ  
. μ μ » , «..  
μ μ ».  
μ ( ) μ ( )  
:  
**9.2:**

	μ 7/8 μ 1".	μ 1, μ 7/8	μ μ 8/8
7/8 μ μ 1.			
	21	0	21
	13	11	24
	34	11	45

μ μ 2 McNemar μ  
μ  
(  $\chi^2 = 12,73$ ,  $df=1$ ,  $p<0,001$ ).  
μ μ , μ μ , μ  
μ μ μ μ  
μ . 5 , ( μ ) μ  
μ μ μ 1/4 ,  
μ « μ 1/3 »  
( μ 1) μ  
μ μ ) ,  
μ .



10:

	1/4 5	3 1/3 4
	31 (68,9%)	20 (44,4%)
	7 (15,6%)	0 (0,0%)

31 (68,9%) 20 (44,4%) 7 (15,6%) 0 (0,0%)

10.1:

1/4 5	1/4	62,2%
1	4 μμ	4,4%
1/4	( μ	2,2%
μμ (1/4)		

28 (62,2%) 1/4 5 2 (4,4%) 4 1 (2,2%) 20 (44,4%) 2 μ 1/3 μ (6 μ ) μ 4 ( 24 μμ ) μ 1/3 μ ( 6 μμ ) 2 μμ μ ( ) μ ( ) :





« $\frac{3}{4} : \frac{1}{8} = \frac{4}{3} \times \frac{1}{8} = \dots$ »

12:

	: $\frac{3}{4} : \frac{1}{8}$	: « $\frac{3}{4} : \frac{1}{8} = \frac{4}{3} \times \frac{1}{8} = \dots$ »
	35 (77,8%)	28 (62,2%)
	2 (4,4%)	9 (20,0%)

12, 35 45 (77,8%) 28 (62,2%) 2 (4,4%) 9 (20,0%)

12.1:

	: $\frac{3}{4} : \frac{1}{8} = \frac{4}{3} \times \frac{1}{8} = \dots$		
	24	11	35
	4	6	10
	28	17	45

McNemar  $\chi^2 = 2,7, df=1, p=0,118$ .

«  $\frac{3}{4}$  :  $\frac{1}{3}$  ;»  
 $\frac{3}{4} - \frac{1}{3} = \dots$

13:  $\frac{3}{4} \times \frac{1}{3}$  : «  $\frac{3}{4} - \frac{1}{3} = \dots$  »

	: $\frac{3}{4} \times \frac{1}{3}$	: « $\frac{3}{4} - \frac{1}{3} = \dots$ »
	23 (51,1%)	15 (33,3%)
	5 (11,1%)	2 (4,4%)

45 (51,1%)  
 (31,1%)  
 (6,7%)  
 (35,6%)  
 16

13.1: -  $\frac{3}{4}$   $\frac{1}{3}$   $\frac{1}{3}$   $\frac{1}{3}$

		$\frac{1}{3}$
: $\frac{3}{4}$ : $\frac{1}{8}$	$\frac{3}{4}$ $\frac{1}{3}$ ,	33,3%
	( $\frac{3}{4}$ ) (0,75: 0,125)	17,3%
	$\frac{3}{4} = \frac{6}{8}$	11,1%
	$\frac{3}{4}$ $\frac{1}{3}$	8,9%
	$\frac{3}{4}$ , $\frac{1}{3}$	6,7%
$\frac{1}{3}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{1}{3}$	33,3%	



( μ μ ) 3 (6,7%) μ  
( . . μ μμ / ). μ  
, , 28 (62,2%)  
μ  
μ . μ , μ  
μ , 10 (22,2%) μ μ  
μ « μ » μ μ  
μ . , 5 (11,1%) μ μ  
. μ  
12 (26,7%)  
μ μ μ .









## 6.

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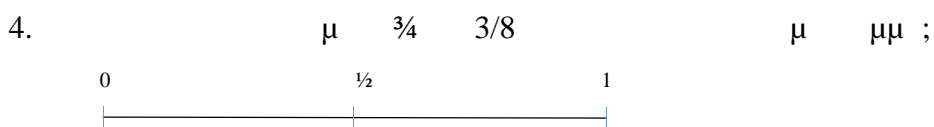
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μ

1.  $3/4 - 1/4 =$   $3 : 0,5 =$   $4 \times 3/4 =$

2.  $3/8 \cdot 7/8$   $2/4 \cdot 4/2$  ;  
 $1/2 \cdot 5/8$   $3/7 \cdot 5/8$

3.  $9/8$  ;  $μ$  ;  $μ$   
4;  $μ$  ,  $μ$   $μ$  ; ;  $μ$  ;  $μ$

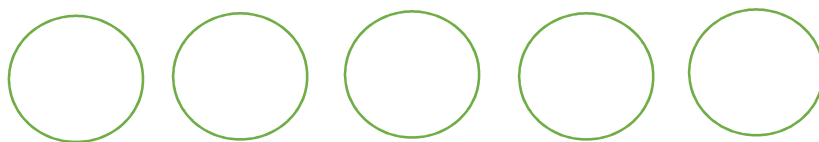


5.  $150\%$   $12$   $90\%$   $40$ .

6.  $μ$   $μ$   $7/8$   $1$ .

6.  $7/8$   $1$   $μ$   $7/8$   $μ$   $8/8$  : «  $μ$   $1$ . »  $μ$   $μ$   
 $μ$  .

7.  $1/4$  .



7 .  $\mu$   $\mu$   $\mu$  :  
 «  $\mu$   $1/3$  »  
 1



$\mu$

8 .  $\mu$   $\mu$   $\mu$   $\mu$  :  
 0,33 0,8 0,242 0,4 0,71

8 .  $\mu$  :  
 $\mu$   $\mu$   $\mu$  :  
 0,53 0,7 0,475 0,12 0,3

$\mu$   $\mu$  :  
 0,3 0,7 0,12 0,53 0,475

$\mu$

9 .  $\mu$   $\mu$   $3/4$   $\mu$   $\mu\mu$  ;  
 $\mu$   $1/8$   $\mu$   $\mu\mu$

9 .  $\mu$   $\mu$   $\mu$  :

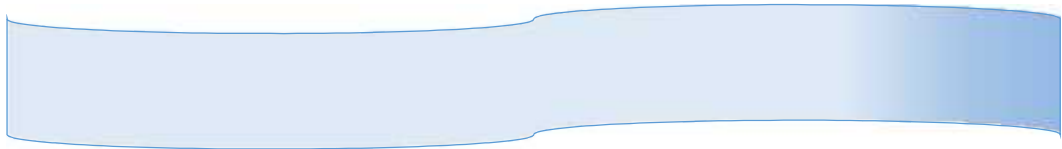
$3/4 : 1/8 = 4/3 \times 1/8 =$

$\mu$

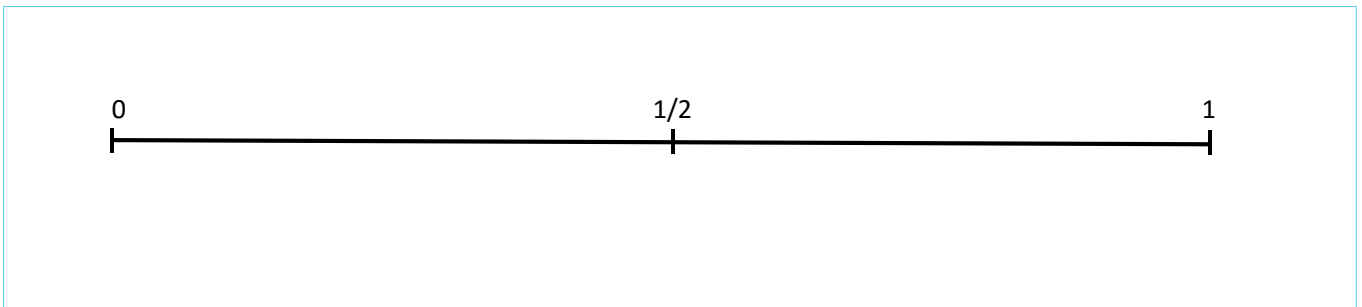


10 .  $\frac{3}{4}$   $\mu$   $\mu$  .  $\mu$   $\frac{1}{3}$   
 ;  $\mu$   $\mu$

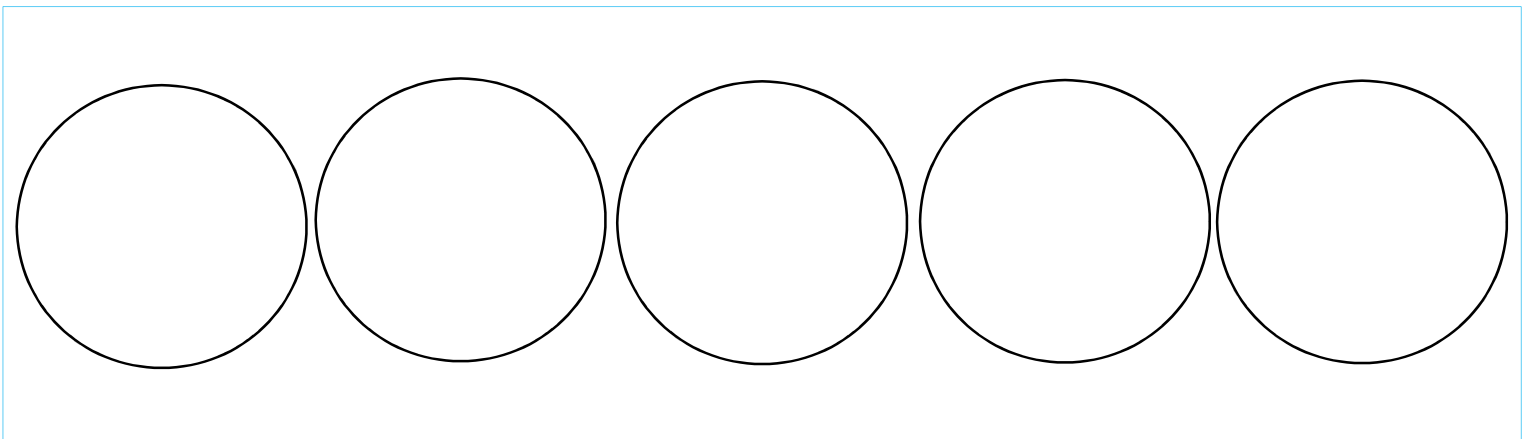
10 .  $\mu$   $\mu$   $\mu$  :  $\mu$   
 $3/4 - 1/3 =$   $\mu$  .



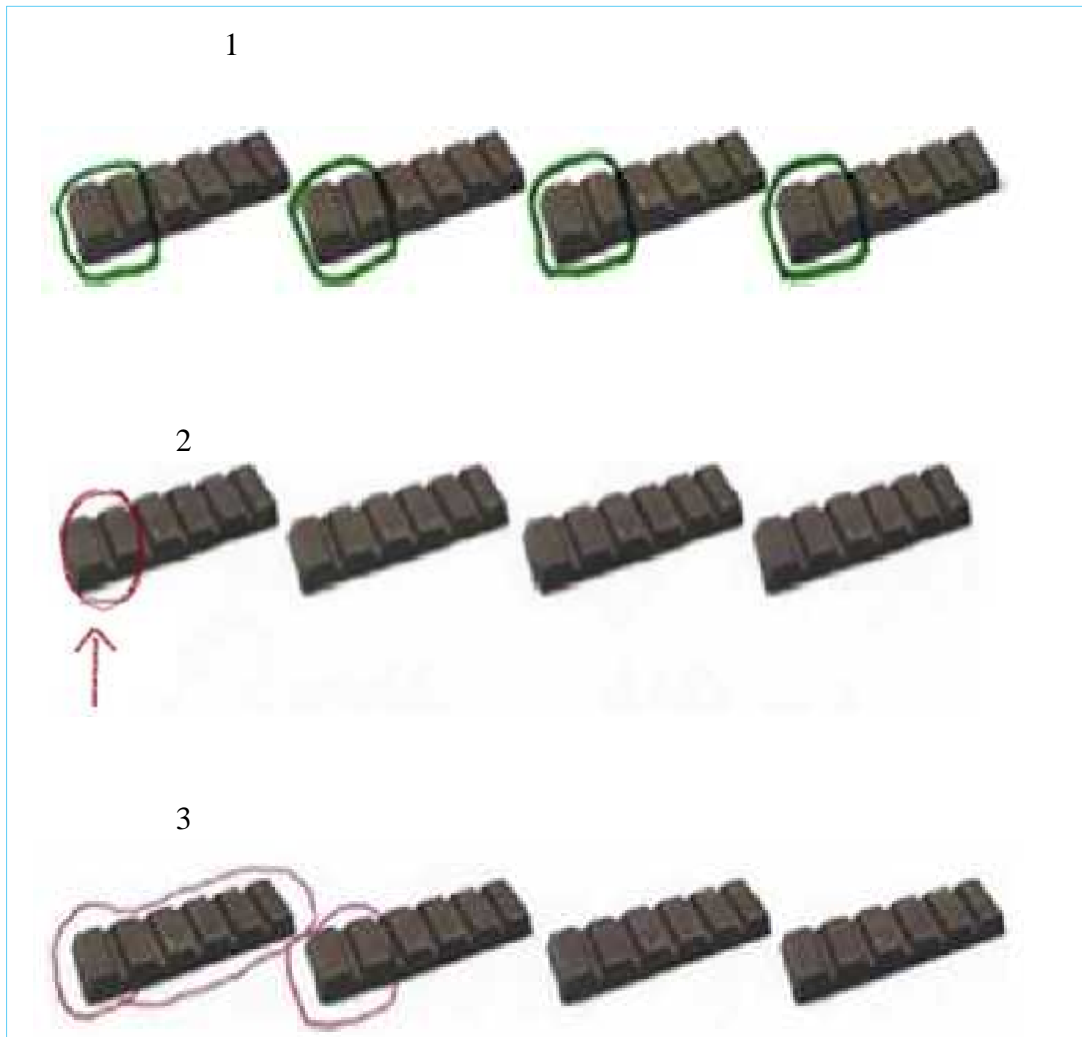
Ερώτηση 4



Ερώτηση 7α



Ερώτηση 7β



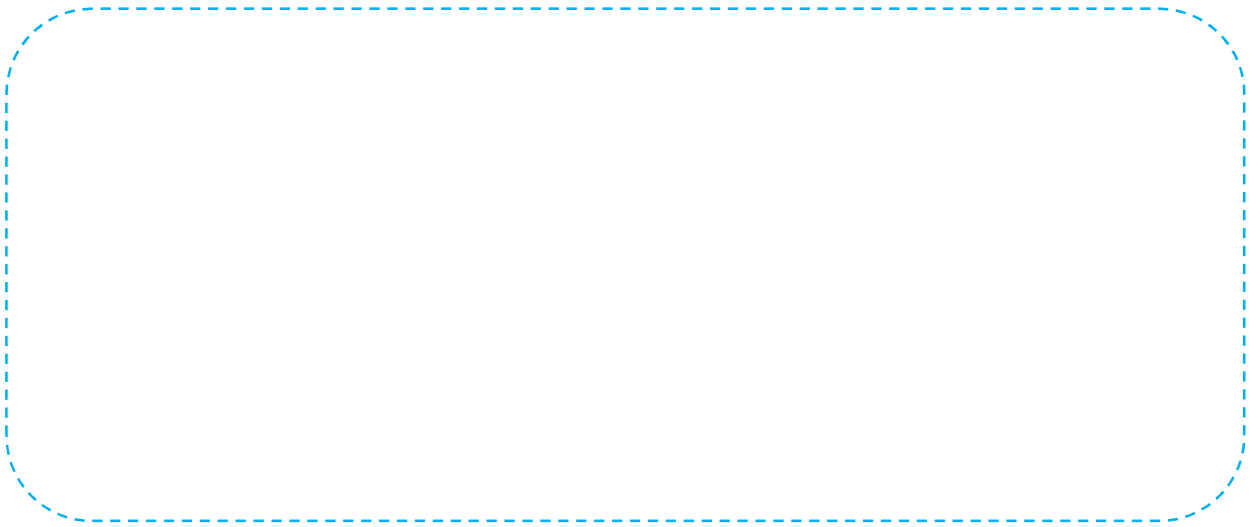
Ερώτηση 8α

0,33 0,8 0,242 0,4 0,71

Ερώτηση 8β

0.3 0.4 0.12 0.53 0.475

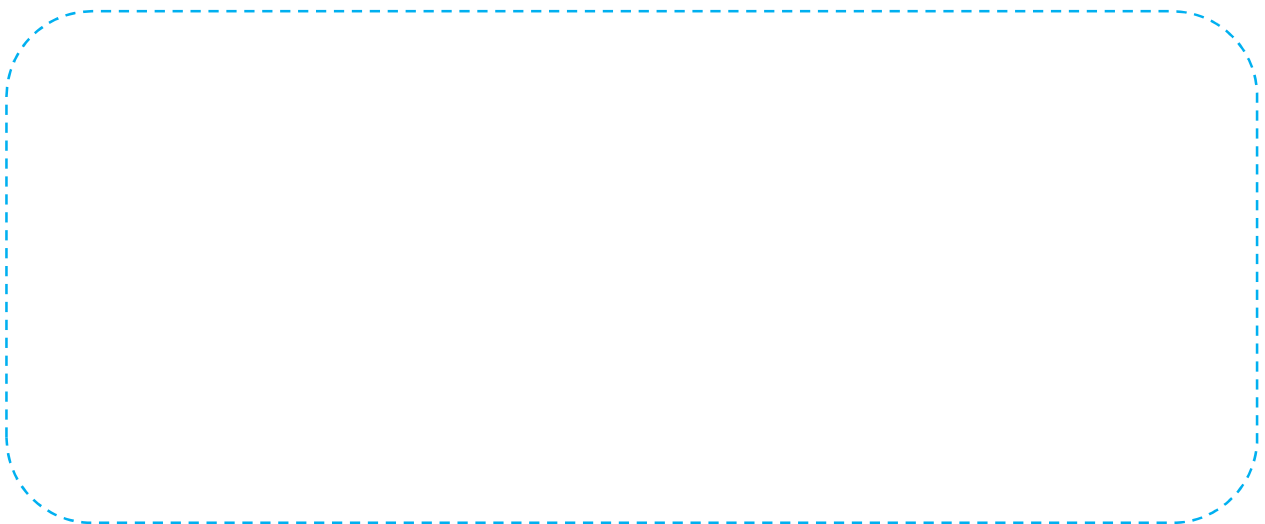
Ερώτηση 9α



Ερώτηση 9β

$$\frac{3}{4} : \frac{1}{8} = \frac{4}{3} \times \frac{1}{8} =$$

Ερώτηση 10α



Ερώτηση 10β

$$\frac{3}{4} - \frac{1}{3} =$$